A NOVEL CIRCULANT APPROXIMATION METHOD FOR FREQUENCY DOMAIN LMMSE EQUALIZATION

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ABSTRACT

The Linear Miminum Mean Square Error (LMMSE) equalizer coefficients of a stationary signal are defined by a Toeplitz system. The Toeplitz structure lends itself to computation in frequency domain, which reduces complexity. In this paper we investigate circulant embedding and circulant approximation methods applied to the preconditioned conjugate gradient (PCG) method and frequency domain equalization. We develop a novel circulant approximation method which improves the performance/complexity tradeoff. All considered algorithms are benchmarked in terms of implementation complexity and capacity achieved by a High Speed Downlink Packet Access (HSDPA) receiver in a multipath fading scenario.

Index Terms— HSDPA, LMMSE, Frequency domain, circulant approximation, FFT, Conjugate Gradient.

1. INTRODUCTION

Signals transmitted over a multipath channel to a mobile receiver are subject to interference and fading. In order to support the data rates required by services such as HSDPA reliably, the equalizer attempts to reverse the effects of the multipath channel. The most common approach is the Linear Minimum Mean Square Error (LMMSE) equalizer. The conjugate gradient method computes the equalizer coefficients with a complexity of $\mathcal{O}(N^2)$, where N is the equalizer length. Alternatively, the coefficients can be computed in frequency domain, which reduces complexity to $\mathcal{O}(N \log N)$. The transformation to frequency domain can only be done at the expense of circulant approximation error, however.

These two approaches have been compared previously to stage time domain and frequency domain equalization against each other [1]. The conjugate gradient method is not mutually exclusive to frequency domain optimizations, however. On the contrary, it can also be reduced to $\mathcal{O}(N \log N)$ complexity and in combination with circulant preconditioners its convergence speed can even be increased.

In Sections 2 and 3 we present the system model and derive the LMMSE equalizer equations. In Section 3.1 we give an overview of FFT optimizations which will be applied in subsequent sections. In Section 3.2 we introduce the classical conjugate gradient method. In Section 3.3 we will then investigate various circulant approximation methods applied to the preconditioned conjugate gradient algorithm. We apply the same methods to frequency domain equalization in Section 3.4 and by combining previous findings for both methods we present a new circulant approximation approach in Section 4. All considered algorithms are benchmarked in terms of implementation complexity and capacity achieved by a High Speed Downlink Packet Access (HSDPA) receiver in a multipath fading scenario.

We use lower-case bold face variables to denote vectors and upper-case bold face variables to denote matrices, $\mathbf{I}_n$ to denote the $n \times n$ unit matrix, $\mathbf{0}_{m,n}$ to denote the $m \times n$ zero matrix. Indices will be omitted where the format is clear from context. We further use $(\cdot)^{\mathrm{T}}$ to denote transposition, $(\cdot)^{\mathrm{H}}$ to denote conjugate transposition, $(\cdot)^*$ to denote complex conjugation, $\mathrm{Re}\, x$ and $\mathrm{Im}\, x$ to denote the real and imaginary parts of x, respectively, $\|\cdot\|_2$ to denote the 2–norm, $\mathrm{E}(\cdot)$ to denote expectation, $\log_2$ to denote the base 2 logarithm, $\log_e$ to denote the base e logarithm, and mod to denote the modulo operation.

2. SYSTEM MODEL

We consider the receive signal

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{v}$$

where

- $\mathbf{y}$ is the $MN \times 1$ receive signal vector,
- $\mathbf{x}$ is the $L+N-1 \times 1$ zero-mean transmit signal vector with unit covariance,
- $\mathbf{v}$ is a $MN \times 1$ zero-mean complex Gaussian noise vector with covariance $\mathbf{I}$,

- M denotes the number of receive antennas,
- N denotes the equalizer length, and
- L denotes the channel length.

Furthermore,

$$\mathbf{H} = [\mathbf{H}_0^{\mathrm{T}}, \mathbf{H}_1^{\mathrm{T}}, \ldots, \mathbf{H}_{M-1}^{\mathrm{T}}]^{\mathrm{T}}$$

is the $MN \times N + L - 1$ channel convolution matrix, where

$$\mathbf{H}_m = \begin{bmatrix} h_{m,0} & h_{m,1} & \cdots & h_{m,L-1} & 0 & \cdots & 0 \\ 0 & h_{m,0} & \cdots & & h_{m,L-1} & \ddots & \vdots \\ \vdots & \ddots & \ddots & & & \ddots & 0 \\ 0 & \cdots & 0 & h_{m,0} & \cdots & \cdots & h_{m,L-1} \end{bmatrix}$$

for $m = 0, \ldots, M-1$ and $h_{m,l}$ denotes the lth delay element of the multipath channel impulse response at antenna m.

The transmit chip sequence is actually a superposition of spread and scrambled UMTS and HSDPA channels. Due to scrambling, the transmit chips are mutually uncorrelated, i.e., $\mathrm{E}(\mathbf{x}\mathbf{x}^{\mathrm{H}}) = \mathbf{I}$. It is therefore possible to disregard the CDMA specific properties of the transmit signal in order to treat equalization entirely at the chip-level.

3. EQUALIZER ALGORITHMS

Let $\mathbf{e}_n$ denote a column vector with a one at index n and zeros otherwise. The desired transmit chip $d = \mathbf{e}_N^{\mathrm{T}}\mathbf{x}$ is estimated using the linear combination of receive samples $\hat{d} = \mathbf{f}^{\mathrm{H}}\mathbf{y}$. The LMMSE equalizer determines $\mathbf{f}$ such that the mean square $J(\mathbf{f}) = \mathrm{E}(|e|^2)$ of the error $e = d - \hat{d}$ is minimized. If the statistics of $\mathbf{y}$ and d are known, this criterion is satisfied by the *Wiener solution* [2]

$$\mathbf{f}_{\mathrm{LMMSE}} = \mathbf{A}^{-1}\mathbf{b} \,, \tag{1}$$

where $\mathbf{A} = \mathrm{E}(\mathbf{y}\mathbf{y}^{\mathrm{H}}) = \mathbf{H}\mathbf{H}^{\mathrm{H}} + \mathbf{I}$ and $\mathbf{b} = \mathrm{E}(\mathbf{y}d^*) = \mathbf{H}\mathbf{e}_N$. The corresponding equalizer is known as the LMMSE equalizer.

Since the channel is not stationary, the LMMSE equalizer processes blocks of receive samples. The number of receive samples per block is chosen small enough so that the channel can be assumed constant. The channel state is estimated for each block and the corresponding LMMSE equalizer weight vector is computed and used to filter the same block of receive samples. The downside of this approach is the need for a channel estimate, as well as the computational complexity involved with solving the system of equations which determines the LMMSE equalizer weight vector.

Note that if $L > N$, the following equalizer algorithms are ignorant of the error due to excess channel length. The auto-correlation and cross-correlation functions are computed for a maximum channel length N. For the investigated multipath channels the resulting computation error is negligible.

3.1. Fast Fourier Transform (FFT)

Using the Fast Fourier Transform, a matrix-vector multiplication can be computed with $\mathcal{O}(N \log_2 N)$ complexity. Let $\mathbf{F}$ be the unitary Vandermonde matrix with elements given by

$$[\mathbf{F}]_{k,l} = \frac{1}{\sqrt{N}} \exp(-i2\pi kl/N) \,, 0 \le k, l < N \,.$$

$\mathbf{F}\mathbf{x}$ is the discrete Fourier transform of an arbitrary vector $\mathbf{x}$. Let $\mathbf{C}$ be a circulant matrix, $\mathbf{c}$ its first column. The columns of $\mathbf{F}^{\mathrm{H}}$ are orthonormal eigenvectors of $\mathbf{C}$. $\mathbf{\Lambda} = \mathbf{F}\mathbf{C}\mathbf{F}^{\mathrm{H}}$ is therefore a diagonal matrix, the entries of which are the Fourier coefficients of $\mathbf{c}$. A matrix-vector multiplication can be computed in frequency domain using the relation $\mathbf{C}\mathbf{f} = \mathbf{F}^{\mathrm{H}}\mathbf{F}\mathbf{C}\mathbf{F}^{\mathrm{H}}\mathbf{F}\mathbf{f} = \mathbf{F}^{\mathrm{H}}\mathbf{\Lambda}\mathbf{F}\mathbf{f}$, which has the complexity of two N–point Fourier transforms, and N multiplications [3]. A fast Fourier transform requires $\frac{1}{2} N \log_2 N$ complex multiplications. If the result is known to be real, the complexity is halved.

Let $\mathbf{E}_{k,l}$ be the $M \times M$ unit matrix with a one at index (k, l) and zeroes otherwise. Let $\mathbf{C}_{k,l}, k, l = 1, \ldots, M$ be $N \times N$ circulant matrices. Then, $\mathbf{C} = \sum_{k,l=1,\ldots,M} \mathbf{C}_{k,l} \otimes \mathbf{E}_{k,l}$ is an $M \times M$ block $N \times N$ circulant matrix, $\tilde{\mathbf{F}} = \mathbf{F}_N \otimes \mathbf{I}_M$ is the corresponding $M \times M$ block $N \times N$ Vandermonde matrix and $\mathbf{\Lambda} = \tilde{\mathbf{F}}\mathbf{C}\tilde{\mathbf{F}}^{\mathrm{H}} = \sum_{k,l=1,\ldots,M} \mathbf{F}_N\mathbf{C}_{k,l}\mathbf{F}_N^{\mathrm{H}} \otimes \mathbf{E}_{k,l} = \sum_{k,l=1,\ldots,M} \mathbf{\Lambda}_{k,l} \otimes \mathbf{E}_{k,l}$ is an $M \times M$ block diagonal matrix.

The matrix-vector multiplication $\mathbf{A}\mathbf{p}$ of a block Toeplitz matrix $\mathbf{A}$ can be computed via a block circulant matrix-vector multiplication

$$\mathbf{C}\mathbf{p}' = \begin{bmatrix} \mathbf{A} & \cdot \\ \cdot & \cdot \end{bmatrix} \begin{bmatrix} \mathbf{p} \\ \mathbf{0} \end{bmatrix} = \begin{bmatrix} \mathbf{A}\mathbf{p} \\ \cdot \end{bmatrix} \,,$$

where the dots are filled such that $\mathbf{C}$ is circulant. This procedure is known as circulant embedding of $\mathbf{A}$ in $\mathbf{C}$. Note that the Fourier coefficients of $\mathbf{C}$ can be obtained by transforming a zero-padded version of $\mathbf{h}_m = [h_{m,0}, h_{m,1}, \ldots, h_{m,N-1}]^{\mathrm{T}}$ to frequency domain and computing cross-correlations pointwise. This requires M $(2N)$–point Fourier transforms and $M^2/2$ (half due to conjugate symmetry) outer products of length $2N$, i.e., a total of $MN \log_2 2N + M^2 N$ complex multiplications.

The runtime complexity of a Hermitian $M \times M$ block $N \times N$ Toeplitz matrix-vector multiplication is therefore $2MN \log_2 2N + 2M^2 N$ complex multiplications, plus $MN \log_2 2N + M^2 N$ complex multiplications for the initial computation of the Fourier coefficients of $\mathbf{C}$.

3.2. Conjugate Gradient Algorithm (CG)

The conjugate gradient algorithm is an iterative method similar to gradient descent, but instead of searching in the direction of residual error, it moves in mutually "conjugate" directions to increase convergence speed. See [4] for further details

Table 1. CG complexity

line	type	multiplications
4	2–norm	$0.5MN$
9	real scalar, complex vector	$0.5MN$
11	matrix, vector	$(MN)^2$
	inner product	$0.5MN$
12	real scalar, complex vector	$0.5MN$
14	real scalar, complex vector	$0.5MN$

on the conjugate gradient method. The conjugate gradient algorithm has been applied to DS-CDMA previously by [5].

The following procedure solves Equation 1.

Algorithm 1 – Conjugate Gradient

Initialization: $\mathbf{f} = \mathbf{0}$
$\mathbf{r} \leftarrow \mathbf{b}$
$k \leftarrow 0$
while $k < N_{\text{iter}}$ and $\|\mathbf{r}\|_2^2 > 0$ **do**
  if $k = 0$ **then**
    $\mathbf{p} \leftarrow \mathbf{r}$
  else
    $\beta \leftarrow \|\mathbf{r}\|_2^2 \,/\, \|\mathbf{r}_{\text{old}}\|_2^2$
    $\mathbf{p} \leftarrow \mathbf{r} + \beta\mathbf{p}$
  end if
  $\alpha \leftarrow \|\mathbf{r}\|_2^2 / (\mathbf{p}^{\text{H}}\mathbf{A}\mathbf{p})$
  $\mathbf{f} \leftarrow \mathbf{f} + \alpha\mathbf{p}$
  $\mathbf{r}_{\text{old}} \leftarrow \mathbf{r}$
  $\mathbf{r} \leftarrow \mathbf{r} - \alpha\mathbf{A}\mathbf{p}$
  $k \leftarrow k + 1$
end while

From Table 1 the total complexity evaluates to $N_{\text{iter}}((MN)^2 + 2.5MN)$ complex multiplications.

If the block-circulant matrix vector multiplication described in the previous section is used, the computational complexity reduces to

$$O_{\text{CG}}(N_{\text{iter}}) = N_{\text{iter}}(2MN\log_2 2N + 2M^2N + 2.5MN) + MN\log_2 2N + M^2N$$

complex multiplications.

3.3. Preconditioned Conjugate Gradient (PCG)

The preconditioned conjugate gradient method is a modified version of the CG algorithm, which uses the Krylov subspaces of a better conditioned approximation of $\mathbf{A}$ in order to increase convergence speed. In this paper we are especially interested in circulant approximations, which can be inverted using the Fourier transform.

Algorithm 2 – Preconditioned Conjugate Gradient

Initialization: $\mathbf{f} = \mathbf{0}$
$\mathbf{r} \leftarrow \mathbf{b}$
$k \leftarrow 0$
while $k < N_{\text{iter}}$ and $\mathbf{r}^{\text{H}}\mathbf{C}^{-1}\mathbf{r} > 0$ **do**
  $\mathbf{z} \leftarrow \mathbf{C}^{-1}\mathbf{r}$
  if $k = 0$ **then**
    $\mathbf{p} \leftarrow \mathbf{r}$
  else
    $\beta \leftarrow \mathbf{r}^{\text{H}}\mathbf{z} / (\mathbf{r}^{\text{H}}\mathbf{z})_{\text{old}}$
    $\mathbf{p} \leftarrow \mathbf{z} + \beta\mathbf{p}$
  end if
  $\alpha \leftarrow \mathbf{r}^{\text{H}}\mathbf{z} / (\mathbf{p}^{\text{H}}\mathbf{A}\mathbf{p})$
  $\mathbf{f} \leftarrow \mathbf{f} + \alpha\mathbf{p}$
  $(\mathbf{r}^{\text{H}}\mathbf{z})_{\text{old}} \leftarrow \mathbf{r}^{\text{H}}\mathbf{z}$
  $\mathbf{r} \leftarrow \mathbf{r} - \alpha\mathbf{A}\mathbf{p}$
  $k \leftarrow k + 1$
end while

Note that $\mathbf{r}^{\text{H}}\mathbf{z} = \mathbf{r}^{\text{H}}\mathbf{C}^{-1}\mathbf{r}$ is real, because $\mathbf{C}^{-1}$ is positive definite. Individual iterations are more complex than plain CG, because the extra system $\mathbf{C}\mathbf{z} = \mathbf{r}$ has to be solved. If $\mathbf{C}$ is chosen circulant, this can be done in frequency domain using the decomposition

$$\mathbf{z} = \mathbf{C}^{-1}\mathbf{r} = \mathbf{F}^{\text{H}}\mathbf{\Lambda}^{-1}\mathbf{F}\mathbf{r}\,. \tag{2}$$

Two well-known preconditioners are Strang's and Chan's preconditioner. Strang's preconditioner is equivalent to circulant embedding of the top-left quarter of $\mathbf{A}$. The elements of its first column are defined by [3]

$$c_{\text{Strang}}(n) = \begin{cases} a(n) & \text{if } 0 \le n < N/2\,, \\ \operatorname{Re} a(n) & \text{if } n = N/2\,, \\ a(n-N) & \text{otherwise.} \end{cases}$$

Note that the element $c(N/2)$ only exists if N is even. In case of circulant embedding $c(N/2)$ is an arbitrary real number. Strang's preconditioner has been shown to minimize the 1–norm distance to $\mathbf{A}$.

Chan's preconditioner, on the other hand, minimizes the F–norm distance to $\mathbf{A}$. The elements of its first column are defined by [3]

$$c_{\text{Chan}}(n) = \begin{cases} a(0) & \text{if } n = 0\,, \\ (1 - \frac{n}{N})a(n) + \frac{n}{N}a(n-N) & \text{otherwise.} \end{cases}$$

Both of these preconditioners preserve the Hermitian property, i.e., $\mathbf{C} = \mathbf{C}^{\text{H}}$ if $\mathbf{A} = \mathbf{A}^{\text{H}}$. In order to generalize these preconditioners to block Toeplitz matrices, they are applied to each block individually.

These preconditioners cannot easily be computed in frequency domain. We therefore have to compute the autocovariance matrix $\mathbf{A}$ and $M^2/2$ (again half due to conjugate

symmetry) complex N–point Fourier transforms. The auto-covariance matrix can be computed by transforming its circularly embedded version back to time domain using $M^2/2$ complex $2N$–point inverse Fourier transforms. This amounts to a total of $M^2/2 \cdot N/2 \log_2 N + M^2/2 \cdot 2N/2 \log_2 2N$ complex multiplications.

The extra complexity is therefore $MN \log_2 N + M^2 N$ complex multiplications per iteration for the computation of $\mathbf{z}$, plus the initial computation of $\mathbf{C}$ and its Fourier coefficients. For the inversion of $\mathbf{\Lambda}$ we estimate $M^3 N$ complex multiplications.

For Strang's preconditioner this gives a total complexity of

$$\begin{aligned} O_{\mathrm{PCG}}^{\mathrm{Strang}}(N_{\mathrm{iter}}) = O_{\mathrm{CG}}(N_{\mathrm{iter}}) + N_{\mathrm{iter}}(MN \log_2 N + M^2 N) \\ + 0.25\, M^2 N \log_2 N + 0.5\, M^2 N \log_2 2N \\ + M^3 N \end{aligned}$$

complex mutliplications. Chan's preconditioner requires an extra $0.25\, M(M+1)(N-1)$ complex multiplications.

$$O_{\mathrm{PCG}}^{\mathrm{Chan}}(N_{\mathrm{iter}}) = O_{\mathrm{PCG}}^{\mathrm{Strang}}(N_{\mathrm{iter}}) + 0.25\, M(M+1)(N-1)$$

3.4. Circulant Approximation (CA)

Using Equation 2 the equalizer coefficients can also be computed in frequency domain. In order to transform the system $\mathbf{Af} = \mathbf{b}$ to frequency domain, however, the auto-correlation matrix must be approximated by a circulant matrix (circulant approximation, CA). The circulant preconditioners are natural candidates for this purpose.

Frequency domain coefficient computation requires the same computations as for the initialization of the PCG algorithm, i.e.,

$$\begin{aligned} O_{\mathrm{CA}}^{\mathrm{Strang}} &= O_{\mathrm{PCG}}^{\mathrm{Strang}}(0)\ , \\ O_{\mathrm{CA}}^{\mathrm{Chan}} &= O_{\mathrm{PCG}}^{\mathrm{Chan}}(0)\ . \end{aligned}$$

4. ALIASING PRECONDITIONER

A significant amount of complexity is due to the computation of the auto-correlation function, its circulant approximation and the Fourier transformation thereof. We therefore propose the novel circulant approximation defined by

$$c_{\mathrm{Alias}}(n) = \begin{cases} a(0) & \text{if } n = 0\ , \\ a(n) + a(n-N) & \text{otherwise.} \end{cases}$$

It is equivalent to Strang's preconditioner if $a(n) = 0$ for all $n \geq N/2$, i.e., if N is at least twice the channel length. Its Fourier transform is the point-wise modulus squared of the Fourier transform of $\mathbf{h}$. This is easy to see from the following identities.

$$\begin{aligned} \mathcal{F}(f)(k) &= \sum_{n=0}^{N-1} f(n) e^{-i2\pi kn/N} \\ a(n) &= \sum_{m=0}^{N-1} f(m+n) g(m)^* \\ c(n) &= \sum_{m=0}^{N-1} f(m + n \bmod N) g^*(m) \\ \mathcal{F}(c)(k) &= \sum_{n=0}^{N-1} \sum_{m=0}^{N-1} f(n + m \bmod N) g^*(m) e^{-i2\pi kn/N} \\ &= \sum_{m=0}^{N-1} \sum_{n'=0}^{N-1} f(n') g^*(m) e^{-i2\pi k(n'-m)/N} \\ &= (\mathcal{F}(f)\mathcal{F}(g)^*)(k) \end{aligned}$$

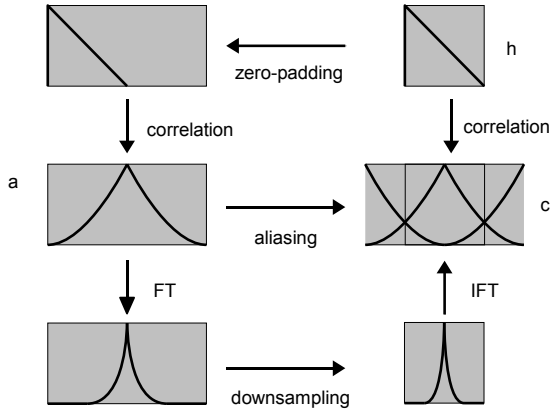


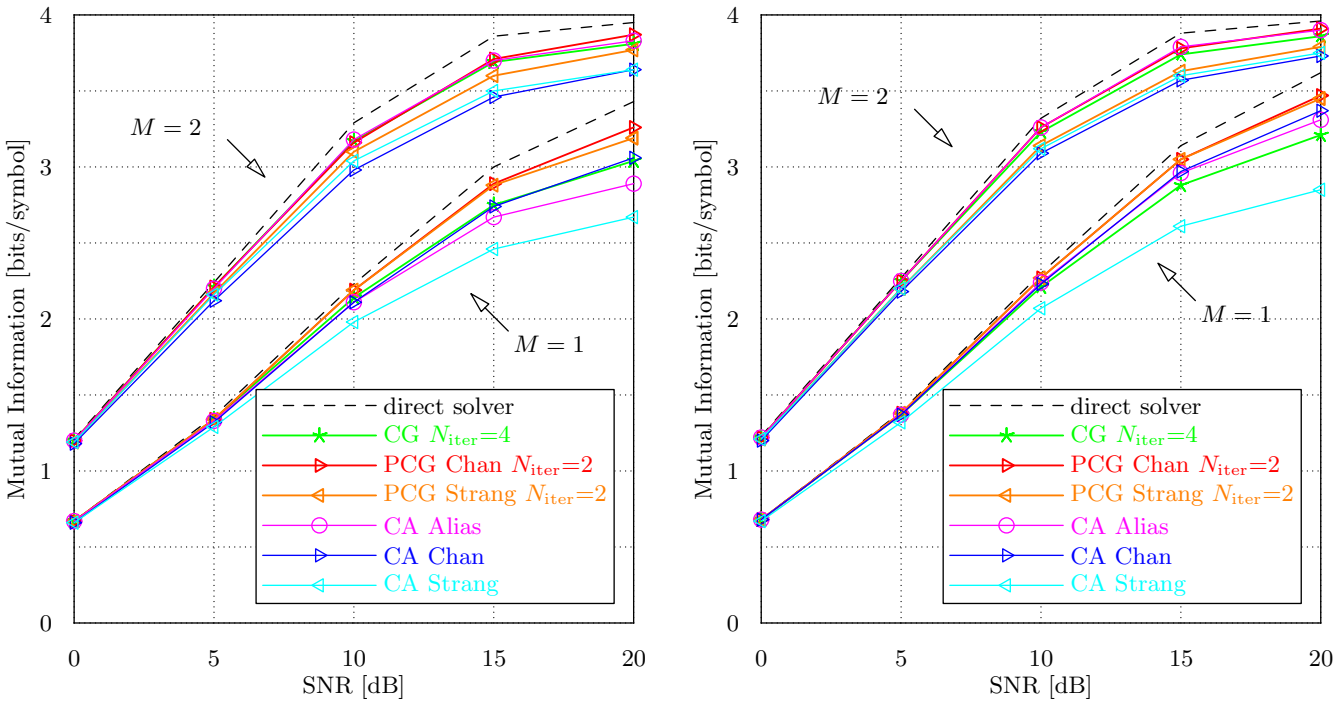
Fig. 1. Exemplary time and frequency domain representations of the aliasing preconditioner

We denote this circulant approximation the *aliasing* preconditioner, because it is constructed analogously to the way downsampling causes aliasing in frequency domain. This "time domain aliasing" effect is illustrated in Figure 1.

Without need for the time-domain autocorrelation function, the computation of the equalizer coefficients for this circulant approximation reduces to M individual N–point Fourier transforms, N outer products of size M, as well as N matrix inversions and matrix-vector multiplications of size M. The total complexity is therefore $O_{\mathrm{CA}}^{\mathrm{Alias}} = MN \log_2 N + M^3 N + 1.5\, M^2 N$ complex multiplications.

5. CONVOLUTION IN FREQUENCY DOMAIN

Note that in each of the CG-based algorithms, the equalizer coefficients can be computed in frequency domain instead, if $\mathbf{f}$ is updated in frequency domain. Since the zero-padded



(a) Pedestrian B

(b) Vehicular A

Fig. 2. Mutual information for $M = 1, 2$ antennas, 16QAM, 8 multicodes, $N = 32$, $P_{\mathrm{HS-PDSCH}} = -3\,\mathrm{dB}$

Table 2. Power delay profiles [6].

Pedestrian B		Vehicular A	
Delay [μs]	**Power** [dB]	**Delay** [μs]	**Power** [dB]
0.0	0.0	0.0	0.0
0.2	-0.9	0.31	-1.0
0.8	-4.9	0.71	-9.0
1.2	-8.0	1.09	-10.0
2.3	-7.8	1.73	-15.0
3.7	-23.9	2.51	-20.0

Fourier transform of the search vector $\mathbf{p}$ is already computed for the matrix-vector multiplication $\mathbf{Ap}$, this comes at no additional complexity. In case of circulant approximation, the coefficients have to be transformed to time domain, padded with zeros, and transformed back to frequency domain.

The complexity of lossless overlap-add frequency-domain convolution is $M \log_2 2N + M^2$ complex multiplications per symbol. Time-domain convolution, on the other hand, requires MN complex multiplications. The latter is clearly more complex for $N \gg M$.

6. SIMULATION RESULTS

Judging from the power delay profiles defined in HSDPA standard testcase specifications [6], typical channels of interest have up to $5\,\mu$s excess delay spread, which at the UMTS chip rate of $3.84\,\mathrm{MHz}$ corresponds to approximately 20 chips. We therefore limit the equalizer length to the next power of 2, i.e., $N = 32$.

The simulation scenario corresponds to the HSDPA standard testcases defined in [6]. Pulse shaping is not modelled, however. Other cell interference is modelled as white Gaussian noise. SNR denotes the average serving cell to interference power ratio.

The HSDPA simulator uses 16QAM modulation, 8 High Speed Physical Downlink Shared Channel (HS-PDSCH) multicodes with spreading factor 16 each, $P_{\mathrm{HS-PDSCH}} = -3\,\mathrm{dB}$ HS-PDSCH power and $P_{\mathrm{CPICH}} = -10\,\mathrm{dB}$ Common Pilot Channel (CPICH) power. The remaining transmit power P_{OCNS} is filled with the Orthogonal Channel Noise Simulator (OCNS) signal, i.e., $P_{\mathrm{HS-PDSCH}} + P_{\mathrm{CPICH}} + P_{\mathrm{OCNS}} = 1$. The Pedestrian B and Vehicular A power delay profiles are reproduced in Table 2. The multipath taps are drawn from a Rayleigh distribution, once for each subframe. The channel is assumed constant over one subframe.

After equalization, the signal is descrambled and despread and log-likelihood ratios (LLRs) are generated using the Max-Log-MAP algorithm. In order to measure performance independently of channel coding, we are interested in the channel capacity of transmitter, multipath channel, and receiver combined. Let X be the BPSK-modulated transmit bit, Y the

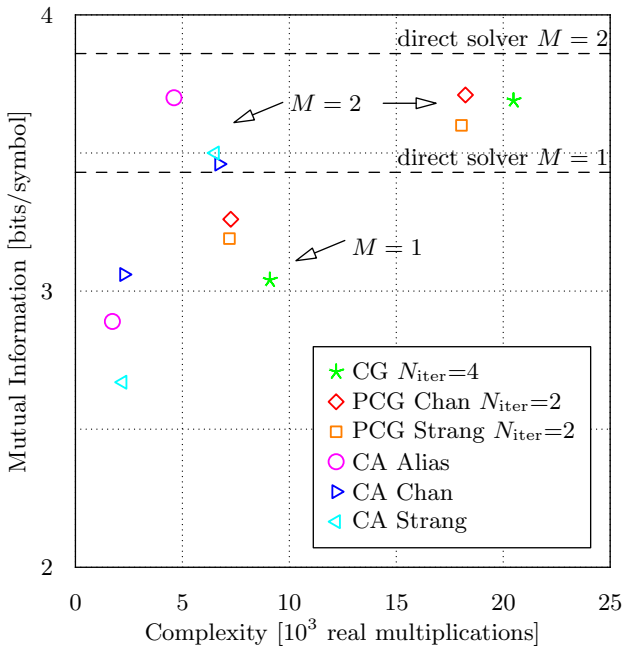


Fig. 3. Pedestrian B, 16QAM H-Set 6, $P_{\text{HS-PDSCH}} = -3\,\text{dB}$, $\text{SNR} = 20\,\text{dB}$ if $M = 1$ antenna, $15\,\text{dB}$ if $M = 2$ antennas.

despreader output,

$$\lambda = \log_e \frac{P\{+1|Y\}}{P\{-1|Y\}}$$

the corresponding LLR and $P\{X|Y\}$ the a posteriori probability distribution as determined by the Max-Log-MAP algorithm. Using the following identities we can express the mutual information between X and Y in terms of λ [7].

$$\begin{aligned} I(X;Y) &= H(X) - H(X|Y) \\ &= 1 - \mathrm{E}\log_2 \frac{1}{P\{X|Y\}} \\ &= 1 - \mathrm{E}\log_2(1+\exp(-X\lambda(Y))) \end{aligned}$$

We estimate the mutual information by substituting expectation with the empirical average

$$1 - \frac{1}{n}\sum_{k=1}^{n} \log_2(1+\exp(-x_k\lambda_k)) \; . \qquad (3)$$

The simulation stop criterion is 1000 subframes and a minimum of 1000 bit errors. The estimates are therefore based on averages over 15360 bits for each of at least 1000 channel realizations.

Performance in terms of mutual information is shown in Figure 2. It can be noted that circulant approximation yields generally worse performance than the conjugate gradient based methods. Strang's preconditioner usually performs worse than Chan's preconditioner. The aliasing preconditioner performs very well for $M = 2$ antennas. The preconditioned conjugate gradient method with Chan's preconditioner always has the best performance.

Based on the above parameters, we can now estimate the implementation complexity for each algorithm. Figure 3 plots performance against complexity for the Pedestrian B power delay profile. The operating points $M = 1, \text{SNR} = 20\,\text{dB}$ and $M = 2, \text{SNR} = 15\,\text{dB}$ were chosen from the performance plots for maximum sensitivity with respect to mutual information. The circulant approximation based methods have a much better performance/complexity tradeoff. For $M = 2$ antennas, the aliasing preconditioner achieves the same performance as the conjugate gradient based methods at less than one third of the computational complexity.

7. CONCLUSION

We have investigated the conjugate gradient method and frequency domain equalization. The conjugate gradient based methods show better performance than the corresponding circulant approximations, but the latter is less complex. We introduced the new *aliasing* preconditioner. If two receive antennas are used it achieves the same performance as the conjugate gradient based methods at one third of the complexity.

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